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Energy Associated with a Charged Regular Black Hole

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Abstract

We show that using an adequate coordinate transformation the
charged regular black hole solution given by Ayón-Beato and Garcia
can be put in the Kerr-Schild form. Then we use this metric in Kerr-
Schild Cartesian coordinates with a result given by Virbhadra and
obtain the energy distribution associated with this.

Keywords: energy distribution, charged regular black hole

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1 INTRODUCTION

A problem which still remains unsolved in general relativity is the energy-
momentum localization.

Although an adequate coordinate-independent expression for energy and
momentum density has not given yet, various energy-momentum complexes
including those of Einstein [1]-[2], Landau and Lifshitz [3], Papapetrou [4],
Bergmann [5], Weinberg [6] and Møller [7] lead to acceptable results for
many space-times. Cooperstock [8] gave his opinion that the energy and
momentum are confined to the regions of non-vanishing energy-momentum
tensor of the matter and all non-gravitational fields. Also, Chang, Nester and

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Chen [9] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum. The above energy-momentum complexes, except that of Møller, need to carry out the calculations in "Cartesian coordinates". The results obtained for some well-known space-times are encouraging [10]-[25].

Aguirregabiria, Chamorro and Virbhadra [15] obtained that several energy-momentum complexes "coincide" for any metric of the Kerr-Schild class. In [19] Virbhadra established that several energy-momentum complexes (ELLPW) comply with the quasi-local mass definition of Penrose for a general non-static spherically symmetric metric of the Kerr-Schild class. He obtained the expression of the energy distribution for this general metric.

In this paper we first show that using an adequate coordinate transformation we can express the charged regular black hole (Ayón-Beato and Garcia, (ABG)) [27] metric in Kerr-Schild Cartesian coordinates. Then we compute the energy distribution in this space-time using the energy expression given by Virbhadra [19]. The result is the same as we obtained in the Einstein prescription [26] using the Schwarzschild Cartesian coordinates. We use the geometrized units ($G = 1, c = 1$) and follow the convention that Latin indices run from 0 to 3.

2 ENERGY OF THE CHARGED REGULAR BLACK HOLE

A solution to the coupled system of the Einstein field and equations of the nonlinear electrodynamics was recently given by E. Ayón-Beato and A. Garcia (ABG) [27]. This is a singularity-free black hole solution with mass M and electric charge q. Also, the metric at large distances behaves as the Reissner-Nordström solution. The usual singularity of the RN solution, at $r = 0$, has been smoothed out and now it simply corresponds to the origin of the spherical coordinates. This solution is given by the line element

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (1)$$

where

$$A(r) = B^{-1}(r) = 1 - \frac{2M}{r}(1 - \tanh(\frac{q^2}{2Mr})). \qquad (2)$$

If the electric charge vanishes we reach the Schwarzschild solution. At large distances (1) resembles to the Reissner-Nordström solution and can be written

$$A(r) = B^{-1}(r) = 1 - \frac{2M}{r} + \frac{q^2}{r^2} - \frac{q^6}{12M^2r^4} + O(\frac{1}{r^6}). \qquad (3)$$

The Kerr-Schild class space-times have the form

$$g_{ik} = \eta_{ik} - Hl_il_k, \qquad (4)$$

where $\eta_{ik} = diag(1, -1, -1, -1)$ is the Minkowski metric. H represents the scalar field and l_i is a null, geodesic and shear free vector field in the Minkowski space-time. We also have

$$\begin{aligned}
\eta^{ab}l_al_b &= 0, \\
\eta^{ab}l_{i,a}l_b &= 0, \\
(l_{a,b} + l_{b,a})l^a{}_{,c}\eta^{bc} - (l^a{}_{,a})^2 &= 0.
\end{aligned} \qquad (5)$$

Aguirregabiria, Chamorro and Virbhadra [15] showed that for the space-times of the Kerr-Schild class the energy-momentum complexes of Einstein, Landau and Lifshitz, Papapetrou and Weinberg "coincide".

We use the transformation

$$u = t + \int A^{-1}(r)dr \qquad (6)$$

and we get

$$dt = du - A^{-1}(r)dr. \qquad (7)$$

Also, we obtain

$$dt^2 = du^2 + A^{-2}(r)dr^2 - 2A^{-1}(r)drdu. \qquad (8)$$

Using (8) the line element (1) becomes

$$ds^2 = A(r)du^2 - 2dudr - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (9)$$

that is the static case of the general non-static spherically symmetric space-time of the Kerr-Schild class used by Virbhadra [19] to calculate the energy distribution with the energy-momentum complexes of Einstein, Landau and Lifshitz, Papapetrou and Weinberg (ELLPW).

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Now, with the transformations

$$
\begin{aligned}
T &= u - r, \\
x &= r \sin\theta \cos\varphi, \\
y &= r \sin\theta \sin\varphi, \\
z &= r \cos\theta,
\end{aligned}
\tag{10}
$$

the metric given by (9) can be written

$$
ds^2 = dT^2 - dx^2 - dy^2 - dz^2 - (1 - A) \times [dT + \frac{xdx + ydy + zdz}{r}]^2. \tag{11}
$$

For (11) we have $H = 1 - A$ and $l_i = (1, \frac{x}{r}, \frac{y}{r}, \frac{z}{r})$. We evaluate the energy using the expression obtained by Virbhadra [19], (see Eq. (31) therein), in the case of a general non-static spherically symmetric space-time of the Kerr-Schild class, and which is the same in the (ELLPW) prescriptions. Our case is the static case, which is a special case of the metric considered by Virbhadra. We get

$$
E(r) = \frac{r}{2}(1 - A(r)). \tag{12}
$$

We obtain for the energy distribution of the ABG black hole

$$
E(r) = M(1 - \tanh(\frac{q^2}{2Mr})) \tag{13}
$$

and

$$
E(r) = M - \frac{q^2}{2r} + \frac{q^6}{24r^3M^2} - \frac{q^{10}}{240M^4r^5} + O(\frac{1}{r^6}). \tag{14}
$$

Also, for (14) we can write

$$
E(r) = E_{RN}(r) + \frac{q^6}{24M^2r^3} - \frac{q^{10}}{240M^4r^5} + O(\frac{1}{r^6}), \tag{15}
$$

where the term $E_{RN}(r)$ represents the energy of the Reissner-Nordström solution that corresponds to the Penrose [28] quasi-local mass definition.

We define $E' = \frac{E(r)}{M}$, $Q = \frac{q}{M}$ and $R = \frac{r}{M}$ and we have $E' = 1 - \tanh(\frac{Q^2}{2R})$. We plot the expression of E' in the Figure 1 (E' on Y-axis is plotted against R on X-axis, for $Q = 0.1, ..., 1$).

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3 DISCUSSION

Bondi [29] gave his opinion that a nonlocalizable form of energy is not admissible in relativity.

For the charged regular black hole solution given by Ayón-Beato and Garcia we make a coordinate transformation that allow us to evaluate the energy distribution using the expression of the energy obtained by Virbhadra [19] (see Eq. (31) therein), for a general non-static spherically symmetric space-time of the Kerr-Schild class. The new form of the (ABG) metric is a special static case of the metric considered by Virbhadra [19]. The energy distribution depends on the mass M of the black hole and electric charge q. Also, the result is the same as we obtained [26] in the Einstein prescription using the Schwarzschild Cartesian coordinates. Our result sustain the viewpoint of Virbhadra [19] that the Einstein energy-momentum complex is the most adequate to evaluate the energy distribution of a given space-time.

Acknowledgments

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